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LETTER TO THE EDITOR

Observation of localisation and interaction corrections to the conductivity and thermopower of a two-dimensional electron gas

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Abstract. We have measured the conductivity and thermopower of silicon-on-sapphire MOSFETs at low temperatures as a function of perpendicular and parallel magnetic fields. The magnitude of the thermopower was found to decrease for perpendicular fields up to about 1 T. An explanation of this effect is offered in terms of the quenching of weak localisation and the dominance of phonon-drag effects. At higher fields, both perpendicular and parallel, the thermopower was found to increase again, which we attribute to enhancement of electron–electron interactions by the field.

It is now well known (e.g. Lee and Ramakrishnan 1985) that quantum interference effects cause localisation of all electron states in a disordered two-dimensional system and that this gives rise to a correction to the Boltzmann conductivity σ_B. For a phase relaxation time τ_φ that varies as T^{-p}, the correction is given in the limit of weak disorder ($k_F l \gg 1$) by

$$\delta\sigma_L = (g_v \alpha p e^2/2\pi^2\hbar)\ln(T/T_0) \tag{1}$$

where g_v is the valley degeneracy and α an inter-valley scattering parameter. A magnetic field B quenches the weak localisation by introducing a phase difference between waves travelling in opposite directions around a scattering loop. The quenching of weak localisation gives rise to a negative magneto-resistance. Hikami *et al* (1980) found the magneto-conductivity for $k_F l \gg 1$ to be

$$\Delta\sigma_L = (g_v \alpha e^2/2\pi^2\hbar)[\Psi(\tfrac{1}{2} + 1/a\tau_\varphi) - \Psi(\tfrac{1}{2} + 1/a\tau) + \ln(\tau_\varphi/\tau)] \tag{2}$$

where $a = 4eBD/\hbar$, τ is the elastic scattering time and Ψ is the digamma function; D is the diffusion constant which can be obtained from σ_B via the Einstein relation.

Electron–electron interactions in a disordered system also give rise to a correction to the conductivity given by Fukuyama (1980) as

$$\delta\sigma_I = \sigma_B g \lambda \ln(4\pi k_B T/\hbar) \tag{3}$$

where k_B is Boltzmann's constant, $\lambda = (\pi k_F l)^{-1}$ is a disorder parameter and $g = g_1 + 2g_2 - (g_3 + g_4)$, g_1 to g_4 being coupling constants corresponding to different types of interaction. In a silicon inversion layer, such an interaction correction is relatively

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small compared with the localisation correction (Uren *et al* 1981). However, a positive magneto-resistance becomes evident at higher fields, arising from suppression of anti-parallel spin interactions when the Zeeman energy $g_L \mu_B B$ becomes comparable to $k_B T$ (g_L is the Landé g-factor and μ_B is the Bohr magneton). Since this is a spin effect, it can be observed when B is parallel to the inversion layer; this is in contrast to the weak localisation correction which depends on the perpendicular component of B (Davies and Pepper 1983). The classical positive magneto-resistance is also absent in a parallel field. Lee and Ramakrishnan (1982, 1985) found the parallel-field interaction magneto-conductivity to be given by

$$\Delta \sigma_I = -(e^2 F^* / 4\pi^2 \hbar) G(b) \tag{4}$$

where $b = g_L \mu_B B / k_B T$. F^* is given by

$$F^* = (8/F)(1 + F/2) \ln(1 + F/2) - 4 \tag{5}$$

which F, being a measure of the screening, is given by

$$F = \int_0^{2\pi} \frac{d\theta}{2\pi} \left(1 + \frac{2k_F}{\kappa} \sin(\tfrac{1}{2}\theta)\right)^{-1}. \tag{6}$$

Here, κ is the inverse screening length in two dimensions, given by

$$\kappa = g_v e^2 m^* / 2\varepsilon\varepsilon_0 \pi \hbar^2. \tag{7}$$

$G(b)$ is a complicated function, recently evaluated for $0.1 < b < 100$ by Burdis and Dean (1988). They found the following limiting forms:

$$G(b) = 0.091 b^2 \qquad b \ll 1 \tag{8a}$$

$$G(b) = \ln(b/1.298) \qquad b \gg 1. \tag{8b}$$

Much work has therefore been done, both experimental and theoretical, on localisation and interaction corrections to the conductivity. This is however only one of the transport parameters of a system. Recently, interest has grown in thermal aspects of transport in a disordered system, including the diffusion thermopower S_d and the thermal conductivity (e.g. Kearney and Butcher 1988). Calculations have been made to obtain the effect of quantum interference on S_d (Ting *et al* 1982, Castellani *et al* 1988) and the more recent work predicts a logarithmic correction to S_d. Ting *et al* (1982) also calculated the interaction correction to S_d and found it to be of a similar form to (3).

In an inversion layer, however, it is difficult to verify these predictions experimentally, since phonon-drag effects dominate the thermopower between 1.2 and 4.2 K (Nicholas 1984, Gallagher *et al* 1987), giving rise to a contribution S_{ph} which varies approximately as T^3. This temperature dependence means that finding a $\ln T$ correction would be very difficult and so we have used a magnetic field to look for a magneto-thermopower at constant temperature.

The samples used in these measurements were low mobility ($0.05\,m^2\,V^{-1}\,s^{-1}$) n-channel MOSFETs fabricated from (100) Si on ($\bar{1}012$) sapphire using an aluminium gate process. The silicon thickness was 0.3 μm on average. The devices were of standard Hall-bar geometry, measuring 2000 μm by 300 μm with an oxide thickness of 2000 Å. Four-terminal measurements of resistance were made using phase sensitive detection in the temperature range 1.2–4.2 K and in magnetic fields up to 7.5 T. The DC measurement of thermopower is described in an earlier paper (Syme *et al* 1989), hereafter referred to as I, and was also used for some of the measurements here. For measuring S in a magnetic

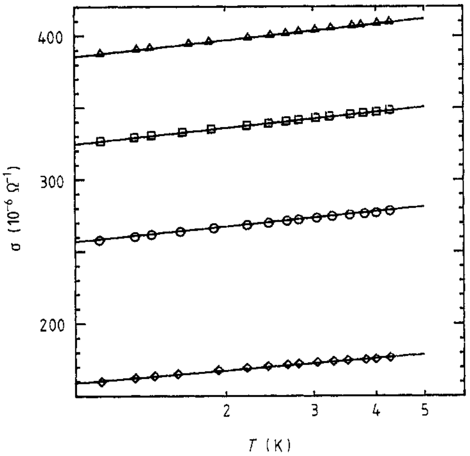

Figure 1. The conductivity of silicon-on-sapphire MOSFETs plotted against temperature for the carrier concentrations indicated in table 1. The full lines are the best fits to $\sigma = A + B \ln T$.

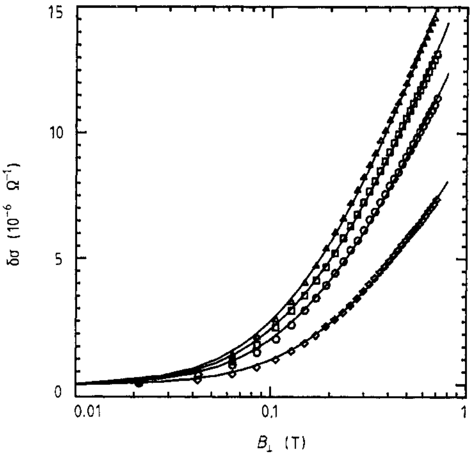

Figure 2. The magneto-conductivity plotted against perpendicular magnetic field for a temperature of 1.9 K. The full curves are fits to equation (2). Symbols as defined in table 1.

Table 1. Explanation of symbols used in figures 1–6. Parameters defined in the text.

Symbol	n (10^{16} m^{-2})	σ_B (10^{-4} Ω^{-1})	$k_F l$	l (10^{-8} m)
◇	2.10	1.59	2.1	0.8
○	3.18	2.57	3.3	1.0
□	4.26	3.25	4.2	1.7
△	5.27	3.86	5.0	1.2

field however, it was found more convenient to use a low-frequency AC technique. An on-chip heater was modulated at frequencies of less than 5 Hz (in order to keep temperature variations along the chip in phase) and the resulting thermoelectric EMF was detected using a low-frequency lock-in technique.

The dependence of the thermopower on temperature and carrier concentration is discussed in I. It should be noted here that the phonon-drag part of the thermopower was dominant ($S_{ph} \gg S_d$). The temperature dependence of the conductivity is shown in figure 1. Table 1 indicates the carrier concentrations used and the corresponding values of $k_F l$. The logarithmic nature of the correction is obvious and fitting the data to equation (1) yields values of $g_v \alpha p$ close to 1, suggesting that $p \simeq 1$.

Figure 2 shows the magneto-conductivity at low magnetic fields for a temperature of 1.9 K. The data have been fitted to equation (2) to obtain values for $g_v \alpha$ and τ_φ. Agreement with the temperature dependence data is found, in that τ_φ varies approximately as T^{-1}, but the values of $g_v \alpha$ extracted are somewhat smaller.

This behaviour was found by Kawaguchi and Kawaji (1982) for samples with $k_F l$ approaching 1, reflecting the fact that (2) is strictly only valid in the limit $k_F l \gg 1$. The values of $k_F l$ for the four carrier concentrations used are listed in table 1: the very low

Table 2. Values of various parameters used in this paper.

n ($10^{16}\,\mathrm{m}^{-2}$)	E_F (meV)	F^*	F	$g_v\alpha$ (measured)	$g_v\alpha$ (inferred)
2.10	6.0	1.46	1.90	0.54	0.79
3.18	9.1	1.12	1.33	0.72	0.83
4.26	15.7	1.09	1.31	0.81	0.85
5.27	20.3	0.69	0.80	0.90	0.87

value found for $g_v\alpha$ at a carrier concentration of $2.10 \times 10^{16}\,\mathrm{m}^{-2}$ may therefore be due to trying to use the theory at too low a value of k_Fl.

However, even for the highest value of k_Fl, $g_v\alpha$ is still less than one and this may be accounted for by electron–electron interactions affecting the magneto-resistance in a perpendicular field as proposed by Fukuyama (1985). He gave the effective value of $g_v\alpha$ in the range of magnetic fields used in this work as

$$g_v\alpha = 1 - \tfrac{1}{2}F/(1 + \tfrac{1}{2}F\ln(1.143E_F/k_BT)). \qquad (9)$$

Paquin *et al* (1988) used this theory to account qualitatively for values of $g_v\alpha < 1$ in uniaxially-stressed (100) Si MOSFETs, although the quantitative agreement was poor, suggesting that interaction effects were much larger than allowed for by the theory. We have calculated values of $g_v\alpha$ from (9) using the values of F obtained from parallel-field magneto-resistance measurements described below; these are presented in table 2. In order to calculate the Fermi energy E_F, the results of Kawaji *et al* (1976) were used to estimate the occupations of the different valleys. The agreement for the higher values of k_Fl is quite good, suggesting that the deviations at low k_Fl may indeed be due to equation (2) beginning to break down.

Figure 3 shows the magneto-conductivity in a large parallel magnetic field. The data have been fitted to equation (4) using the high-field limit of $G(b)$. Values of F^* have been extracted and are shown in table 2. The values are high (theoretically, $F < 1$ which means $F^* < 0.85$), but this has been observed before by Bishop *et al* (1982). These authors also found a fall off in F^* with increasing carrier concentration at high gate voltages, in agreement with our results. This was qualitatively explained by an increase in k_F causing a reduction in F from equation (6) and hence a reduction in F^*. The reason for the high values of F^* is still uncertain.

The remaining results are concerned with the first observations of localisation and interaction corrections to the thermopower. Figure 4 shows the magneto-thermopower in a perpendicular magnetic field. The negative values of $\delta S/S$ can be accounted for by a quantum-interference correction in the presence of phonon drag. The electric current may be written

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + e^{-1}\boldsymbol{\nabla\mu}) - (\eta/T)\boldsymbol{\nabla T} \qquad (10)$$

where μ is the chemical potential, $\boldsymbol{E}$ is the electric field and σ and η are linear kinetic coefficients; σ can be identified with the electrical conductivity. If $\boldsymbol{J} = 0$ and the other parameters are non-zero, then (10) gives

$$\eta/\sigma T = |\boldsymbol{E} + e^{-1}\boldsymbol{\nabla T}|/|\boldsymbol{\nabla T}| = S. \qquad (11)$$

As stated above, the thermopower S consists of two parts: a phonon-drag part S_{ph} and a diffusion part S_d. In a similar way, η may be split into two parts:

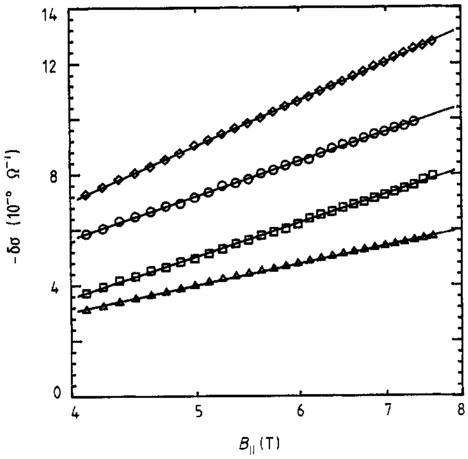

Figure 3. The magnetoconductivity plotted against parallel magnetic field for a temperature of 1.9 K. The full lines are fits to the high field limit of equation (4). Symbols as defined in table 1.

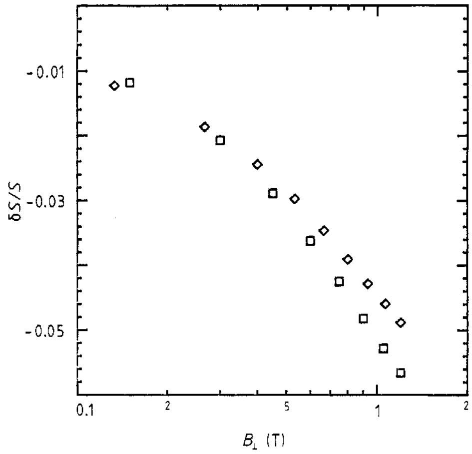

Figure 4. The relative change in the thermopower plotted against perpendicular magnetic field for a temperature of 1.9 K. Symbols as defined in table 1.

$$S = S_{\mathrm{ph}} + S_{\mathrm{d}} = (\eta_{\mathrm{ph}} + \eta_{\mathrm{d}})/\sigma T. \tag{12}$$

A small change δS in the thermopower may thus be expressed in terms of changes in σ and η, namely

$$\delta S/S = \delta\eta_{\mathrm{ph}}/\eta + \delta\eta_{\mathrm{d}}/\eta - \delta\sigma/\sigma. \tag{13}$$

Castellani *et al* (1988) showed that $\delta\eta_{\mathrm{d}} = 0$, although even if $\delta\eta_{\mathrm{d}}$ were non-zero, we still have $\delta\eta_{\mathrm{d}}/\eta \ll \delta\sigma/\sigma$ since $\eta \simeq \eta_{\mathrm{ph}} \gg \eta_{\mathrm{d}}$. We also expect $\delta\eta_{\mathrm{ph}}$ to be zero, because for phonon drag to occur, there must be an inelastic electron–phonon scattering event which breaks the phase coherence required for weak localisation. Thus we predict that in the weak localisation regime, $\delta S/S = -\delta\sigma/\sigma$. Figure 5 shows the ratio of $\delta\sigma/\sigma$ to $\delta S/S$ for two carrier concentrations, obtained using the data in figures 2 and 4. The fact that all the points lie near -1 is good evidence for the validity of the above simple theory.

In addition, we have observed an increase in $\delta S/S$ at higher magnetic fields. Figure 6 shows the magneto-thermopower in a parallel field for two carrier concentrations. Since $\delta S/S$ was also found to increase in high perpendicular magnetic fields, we attribute the increase to a spin-dependent electron–electron interaction effect, similar in origin to that which is responsible for the positive magneto-resistance shown in figure 3. The difference in magnetic field at which the correction seems to start may be due to variations in enhancement of the Landé g-factor (see Englert *et al* 1980).

A similar argument to that presented for weak localisation effects may be used to show $\delta S/S = -\delta\sigma/\sigma$: as before $\delta\eta_{\mathrm{d}}/\eta \ll \delta\sigma/\sigma$. Also the electron–phonon interaction is not expected to contribute to the electron–electron interactions, the latter being of the screened-Coulomb type, and so $\delta\eta_{\mathrm{ph}} = 0$. In figure 7 we show the ratio of $\delta\sigma/\sigma$ to $\delta S/S$ for two carrier concentrations, obtained using the data in figures 3 and 6. Again, the values are close to -1, in agreement with our predictions. The apparent convergence of the values towards -1 for the higher fields suggests that there may be a non-zero $\delta\eta_{\mathrm{ph}}$ for the interaction correction, but that it is nonetheless small.

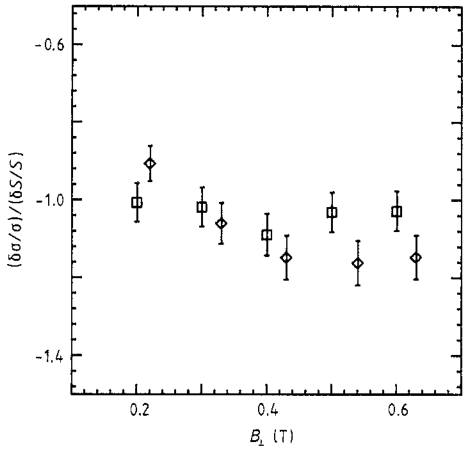

Figure 5. The ratio of the relative changes in the thermopower and conductivity plotted against perpendicular magnetic field for two carrier concentrations, using the data in figures 2 and 4.

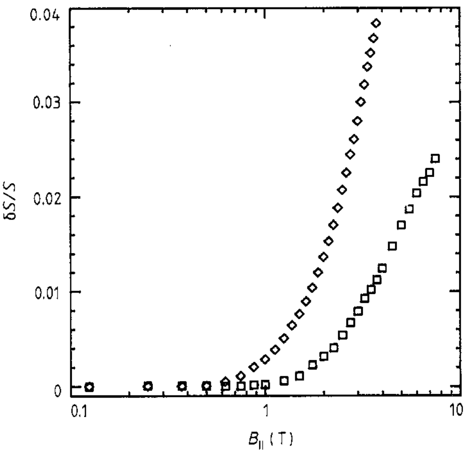

Figure 6. The relative change in the thermopower plotted against parallel magnetic field for two carrier concentrations at a temperature of 1.8 K. Symbols as defined in table 1.

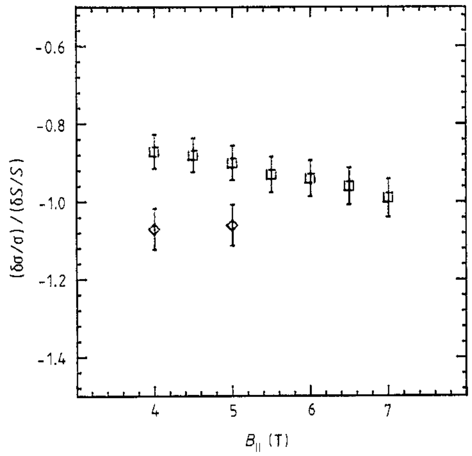

Figure 7. The ratio of the relative changes in the thermopower and conductivity plotted against parallel magnetic field, for the data in figures 3 and 5.

In conclusion, we have observed localisation and interaction corrections in the magneto-conductivity of a silicon-on-sapphire MOSFET and obtained results in agreement with other workers. We have also presented the first observations of these effects in the magneto-thermopower and shown them to be similar to the magneto-conductivity corrections in the presence of phonon drag.

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